Performance Improvement of Activated Sludge Wastewater Treatment by Nonlinear Natural Oscillations

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The paper describes a novel operation strategy for improvement in the performance of biochemical reactors. The dynamic behavior of two coupled continuous stirred-tank reactors with recycle is studied when one of the reactors is being operated under limit cycle regimes producing self-sustained natural oscillations. The periodic output from one reactor is then used as forced input into the other reactor. The novel operation strategy is applied to wastewater treatment by the activated sludge process. It was observed and demonstrated through computation that the overall performance of the system can be enhanced in terms of time-averaged conversion by employing the above-mentioned operation strategy for a particular combination of reactor volume, recycle ratio and other operating parameters. The most significant criterion of this operation strategy lies in the fact that the performance enhancement of the overall system is achieved through natural oscillation rather than forced oscillation. In experimental studies and in eventual application to industrial processes this new concept of coupling free and forced oscillation is advantageous, as it does not require any additional external energy.

1 Introduction

Most chemical processes are designed to operate at a steady-state condition. However, it is well known that for some processes, steady-state operation does not always give the best results and at times, unsteady-state operation improves the overall performance. In general, the average value of the performance of a process operating as unsteady state is not the same as the steady-state operating value. The performance may improve or deteriorate as a result of the unsteady operation. Numerous investigations, both theoretical and experimental studies, have shown that periodic operation of chemical reactors leads to improved reactor performance by producing more reaction products or a more valuable product distribution than a steady-state reactor operation. In this paper, we seek to study the dynamic behavior of operation of the biological wastewater treatment process by activated sludge and whether improved performance can be obtained by operating the reactor as a chemical oscillator.

The periodic operation of chemical reactors through various forms of oscillations to improve reactor performances has been a popular topic of research for more than three decades [1]. For example, experimental studies demonstrated that concentration cycling of catalytic reactors could lead to increased production rates, improved selectivity, an enhanced activity of the catalyst [2,3]. A theoretical investigation [4] followed by an experimental study [5] of the diethyl adipate saponification in a CSTR demonstrated that it is possible to increase the yield through cycling of the feed composition. Theoretical investigations have shown that for many types of homogeneous reactions in a CSTR, an improved yield results from a periodic change in the concentration and/or temperature of the system [6].

The present work differs from the past work in two perspectives. Firstly, all the past efforts have focused on the use of externally forced oscillations in chemical systems, although the process concept involving oscillations appears to be more applicable to biological systems that have a penchant for periodicity. Secondly, most of the investigations studied the possibility of enhancing the performance by inducing periodic forced oscillations in the feed compositions, feed flow rate or reaction temperature. Most of the work on periodic and aperiodic (chaotic) oscillations has focused on understanding the fascinating and exotic behaviors of the system. It appears that little effort has been made until now [7] to improve the reactor performance through natural oscillations. Chemical reactions and chemical reactors exhibit a wealth of dynamic behavior patterns that range from steady-state multiplicity to self-generated and self-sustained natural oscillatory behavior (also known as limit cycle) to quasi-periodic oscillations, multiple limit cycles and transitions to chaos [8]. Local bifurcation theory has been successfully applied to the study of transitions between such patterns in the parameter space [9]. In this work, an operation strategy that involves the use of self-generated natural (or free) oscillation to improve the performance of bioreactors is proposed and applied to biological wastewater treatment by the activated sludge process. Process parameters are chosen such that limit cycles occur within the reactor. The most significant criteria of this operation strategy lie, however, more in the fact that the performance enhancement of the overall system is achieved through a self-generated, self-sustained natural oscillation rather than forced oscillation, which would have required extra external energy. In experimental studies and in eventual application to industrial processes this system will be advantageous in that no externally imposed oscillation is required to generate this situation.

In general, it is difficult to produce oscillations, as they involve high operating costs. In such cases, it will be cost-effective if one could generate self-sustained natural oscillations. Many biological systems exhibit self-generated oscilla-

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tions for certain ranges of parameter values. Therefore, it is possible to create natural oscillations by selecting appropriate process parameters. It is well recognized that in many bioprocesses, particularly those associated with the production of secondary metabolites, two CSTRs connected in series are inherently superior to one CSTR of equal volume. For such a system, it is relatively easy to find a range of operating variables such that a limit cycle takes place naturally in one of the two reactors. For two CSTRs in series with a recycle system, the free oscillation from one CSTR then causes a forced oscillation to occur in the other CSTR by virtue of direct overflow or through recycle, resulting in a possible overall performance improvement for the system when compared with the equivalent system without oscillation.

2 Wastewater Treatment by Activated Sludge Process

The activated sludge process is a continuous or semi-continuous aerobic method for biological wastewater treatment. It includes carbonaceous oxidation and nitrification. The process is based on the aeration of waste water with flocculating biological growth, followed by the separation of treated waste water from biological growth. Part of this growth is then wasted, and the remainder is returned to the system. Usually, the separation of the biomass from the treated waste water is performed by settling (gravity separation) but it may also be done by flotation and other methods [10].

There exist many mathematical models for activated sludge wastewater treatment processes, that range from simple [11–13] to multicomponent multispecies models [14–19] to more complex models [20,21]. The model used in this work is primarily based on the reported work of Curds [14,15] with some modifications discussed below.

The activated sludge wastewater treatment process consists of living microorganisms plus organic matter in an oxygen-rich (aerobic) environment. Microorganisms utilize complex organics as a food source to produce more microorganisms that are eventually settled out of the wastewater. The two basic types of microorganisms important to the operation of an activated sludge system are plants and animals. Plants include bacteria, algae, and fungi. The bacteria are the most important and are primarily responsible for the removal of organic substances from wastewater. Animals include larger microorganisms, such as protozoa, crustaceans, and rotifers. The animals feed on dispersed bacteria that do not settle well and therefore, help polish the quality of the treated effluent. The microorganism population of activated sludge is dynamic in nature. Competition for soluble food occurs among the bacteria, fungi, algae and protozoa. However, most of the theoretical considerations of continuous culture systems have dealt with pure cultures of single organisms, although sewage treatment processes contain a wide variety of organisms. The theoretical and experimental work of Curds [14,15,22] has shown that when bacteria and protozoa are grown together in a reactor as is the situation in the activated sludge process, steady-state conditions do not always exist, instead a series of predator-prey oscillations is observed.

The present study is concerned with the development of a mathematical model that differs from models in the literature, as it recognizes the fact that activated sludge does not consist of a single bacterial species but consists of a variety of microorganisms each with its own particular substrate and mode of life. It is important that the correct kinetic models are used in determining whether periodic operation is advantageous over classical operation, since the use of wrong kinetic models may lead to erroneous conclusions. One might expect that these findings could lead to industrial applications, even though at present industrial applications of periodic operation are rarely found.

3 Mathematical Model

The theory of the continuous culture of bacteria growing in a completely mixed reactor vessel was first described by Monod. The model developed relied on the well-established fundamental microbiological relationships between the specific growth rate of a bacterium and the concentration of an essential growth substance. The specific growth rate, μ, of an organism is related to the concentrations of their limiting substrate by the well-known Michaelis-Menton equation[1)]

$$\mu = \frac{\mu_m S}{K_S + S} \tag{1}$$

where μ and μ_m are the specific growth rate and maximum specific growth rate of the organism, S is the concentration of the substrate, and K_s is the saturation constant which is numerically equal to the substrate concentration when $\mu = \mu_m/2$.

Pure culture growth models of Monod describing the dynamic behavior of the two variables, the concentration of the substrate, S, and the concentration of the bacteria, B, is given by

$$\frac{dB}{dt} = \mu\, B = \frac{\mu_m S B}{K_S + S} \tag{2}$$

$$\frac{dS}{dt} = -\frac{\mu\, B}{Y} = -\frac{\mu_m S B}{Y\left[K_S + S\right]} \tag{3}$$

where Y is the microorganism yield constant.

In reality, complex biological and physicochemical processes occur. In the model considered in this work, two types of bacteria, sludge bacteria, X, and sewage bacteria, B, are considered. Sludge bacteria are the organisms that always flocculate into large settling masses and utilize the soluble substrates of sewage as the feed source. These organisms, however, are not found in the feed sewage, whereas the

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[1)] List of symbols at the end of the paper.

sewage bacteria are born in sewage in considerable quantities and they do not flocculate on entry into the reactor. They remain in suspension in the settling tank and are evenly dispersed throughout the recycled flow and effluent flow. These bacteria also utilize the soluble components of sewage as a food source like the sludge bacteria, but as they remain dispersed throughout, they make themselves available as a food source for other organisms that may be present.

Though protozoa are not present in significant numbers in the sewage, they have a considerable effect on the effluent quality. Three types of protozoa are considered in the model: (a) those which swim freely in the mixed liquor, H, (b) those which crawl over the sludge flocs, P, and (c) those which are directly attached to the sludge floc, P, by means of a stalk. A free-swimming protozoan is usually washed out in the effluent, whereas the protozoan directly attached with the sludge floc will be concentrated and recycled back to the reactor. Protozoa usually use the dispersed bacterial population present in the system as a food source and not on the flocculated bacterial masses. The absolute magnitude of the sludge bacteria and protozoa in the mixed liquor are not of prime importance in the production of a quality effluent. It is the growth rate of the sludge bacteria (controlled by the sludge sewage rate) that determines the concentration of substrate in the effluent, and the growth rate of the protozoa (controlled by the wastage rate in the case of attached protozoa and the dilution rate in the case of free-swimming protozoa) which determines the concentration of dispersed bacteria in the effluent.

The schematic flow diagram used in the model is shown in Fig. 1. The microbial population in the settling tank concentrates by a factor “b”. Some sludge is continuously wasted at a rate F_W, and the remainder is recycled back into the first reactor at a rate F_r. The mathematical model representing the system consists of ten equations, five in each reactor. The mass balance equations in each reactor is given by

Reactor 1:

$$V_1\frac{dS_1}{dt} = FS_o + RFS_2 - (1+R)FS_1 - V_1\frac{\mu_{x(S_1)X_1}}{Y_x} - V_1\frac{\mu_{B(S_1)B_1}}{Y_B} \tag{4}$$

$$V_1\frac{dX_1}{dt} = FX_o + RFbX_2 - (1+R)FX_1 + V_1\ \mu_x(S_1)X_1 \tag{5}$$

$$V_1\frac{dB_1}{dt} = FB_o + RFB_2 - (1+R)FB_1 + V_1\mu_B(S_1)B_1 - V_1\frac{\mu_{P(B_1)C_1}}{Y_C} \tag{6}$$

$$V_1\frac{dH_1}{dt} = FH_o + RFH_2 - (1+R)FH_1 + V_1\ \mu_H(B_1)H_1 \tag{7}$$

$$V_1\frac{dP_1}{dt} = FP_o + RFbP_2 - (1+R)FP_1 + V_1\ \mu_P(Z_1)P_1 \tag{8}$$

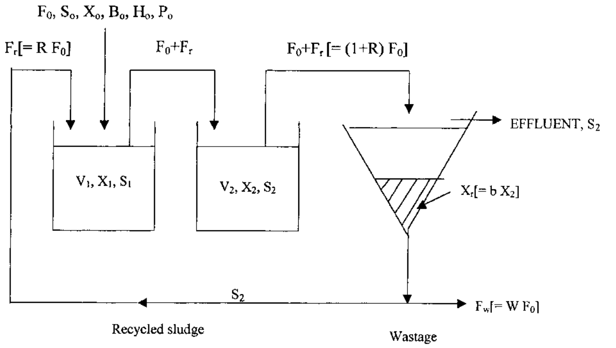

Figure 1. Schematic diagram of activated-sludge plant with two completely mixed reactors in series.

Reactor 2:

$$V_2\frac{dS_2}{dt} = (1+R)FS_1 - (1+R)FS_2 - V_2\frac{\mu_{x(S_2)X_2}}{Y_x} - V_2\frac{\mu_{B(S_2)B_2}}{Y_B} \tag{9}$$

$$V_2\frac{dX_2}{dt} = (1+R)FX_1 - (1+R)FX_2 + V_2\ \mu_x(S_2)X_2 \tag{10}$$

$$V_2\frac{dB_2}{dt} = (1+R)FB_1 - (1+R)FB_2 + V_2\mu_B(S_2)B_2 - V_2\frac{\mu_{P(B_2)C_2}}{Y_C} \tag{11}$$

$$V_2\frac{dH_2}{dt} = (1+R)FH_1 - (1+R)FH_2 + V_2\ \mu_H(B_2)H_2 \tag{12}$$

$$V_2\frac{dP_2}{dt} = (1+R)FP_1 - (1+R)FP_2 + V_2\ \mu_P(Z_2)P_2 \tag{13}$$

where C_i is the total concentration of protozoa in the *i*th reactor, μ_i and Y_i are the specific growth rate and yield constant of the *i*th species, and subscript ‘o’ indicates the initial concentration of species in the entering sewage. The specific growth rate for any organism y is related to their limiting substrate z in reactor i by the Michaelis-Menton equation

$$\mu_y(z_i) = \frac{\mu_{my} z_i}{K_y + z_i} \tag{14}$$

where K_y and μ_{my} are the saturation constant and the maximum growth rate of organism y. The kinetic constants and feed concentrations are given in Tab. 1.

4 Theory of Limit Cycle

The model equations are in the form of

$$d\mathbf{x}/dt = \mathbf{f}(\mathbf{x}, \sigma, \mathbf{p}) \tag{15}$$

where **x** is a vector of the state variables (e.g., conversion, temperature, etc.), σ is a distinguished bifurcation parameter,

Table 1. Kinetic constants of organisms and feed compositions.

Organisms	Concentration in sewage (mg/l)	Maximum specific growth rate (μ_m (h^{-1}))	Saturation constant, K (mg/l)	Yield coefficient, Y
B	30	0.5	10.0	0.5
X	0.1	0.3	15.0	0.5
H	0.1	0.35	12.0	0.5
P	0.1	0.35	12.0	0.5

and **p** is a vector of additional operating parameters. If one defines $\mathbf{y} = \mathbf{x} - \mathbf{x}_s$, where $\mathbf{x}_s$ is the steady-state value of **x**, then Eq. (15) can be written as

$$d\mathbf{y}/dt = \mathbf{Ay} + \mathbf{F}(\mathbf{y}, \sigma, \mathbf{p}) \quad (16)$$

with the Jacobian matrix **A** (σ, **p**) is given by

$$\mathbf{A}\,(\sigma, \mathbf{p}) = \mathbf{D}\,\mathbf{f}(\mathbf{x}_s, \sigma, \mathbf{p}) \quad (17)$$

The dynamic solution and stability of the above equation depends on the eigenvalues of the matrix **A**. A limit cycle or self-sustained natural oscillations are generated when for some proper choice of parameters, σ and **p**, **A** has at least one pair of pure imaginary eigenvalues. For stability, of course, the real part of all other eigenvalues (apart from the one which is purely imaginary) of **A** must be negative. Then, the steady-state solution will be stable but periodic (or oscillatory). Stability of solutions was checked, and it was observed that with time the solution reaches oscillatory steady state (see Fig. 2) when limit cycles exist. Fig. 2 implies that all the eigenvalues of the system have negative real part and only one of the eigenvalues is purely imaginary. This is because if a system has n eigenvalues (number of eigenvalues is equal to the number of equations), and if in general eigenvalues are $\lambda_i = a_i + i\,b_i$, then the overall solution is given by

$$y = \sum_{i=1}^{n} e^{a_i t}\left[C_i \sin b_i t + D_i \cos b_i t\right] \quad (18)$$

where C_i and D_i are constants determined from the initial conditions. Therefore, **y** will approach zero when t goes to $\propto$ if all $a_i < 0$, i.e., $\mathrm{Re}(\lambda_i) < 0$, and **y** will approach $\propto$ when t goes to $\propto$ even if just one eigenvalue has positive real part, say $a_j > 0$, i.e., $\mathrm{Re}(\lambda_j) > 0$. In this work, it was always ascertained that self-sustained natural oscillation was attained, thereafter the average value was evaluated. Therefore, all the results reported in this work indeed represent stable points. The operating region (steady state or oscillatory state) was determined for the two reactors in series with recycle. For different combination of process parameters, either steady state or oscillatory state occurs. Once the two regions were found, it was determined whether natural oscillation or steady state can obtain performance improvement.

5 Numerical Methods

Commercial software TRAX was used for viewing pictures of phase space of the dynamical systems of Eqs. (4) to (13) and investigating their dynamic properties and characteristics. From the pictures of phase space obtained from TRAX, one can identify the existence of limit cycles. The numerical solutions of Eqs. (4) to (13) are then obtained by using the numerical software package LSODE (DSS/2, Differential Systems Simulator, library routine), an initial value integrator program for stiff ordinary differential equations. For better accuracy of the average value, a numerical program was written which first evaluates the maximum and minimum values of a particular variable with time, compares the successive values of maximum (or minimum) to find out that a limit cycle has been established, and then calculates the time-average value of overall conversion from the maximum (or minimum) where the limit cycle is established to the last maximum (or minimum) by using numerical integration techniques of combination of Simpson's one-third and three-eighths method. The steady-state solutions were determined by setting the left-hand side (LHS) of Eqs. (4) to (13) to zero and solving the resulting system of nonlinear equations using a subroutine taken from the CD-ROM [23]. It solves the system of nonlinear equations using a variation of Newton's method. The subroutine evaluates the Jacobian numerically and then uses a forward difference method to find the solutions.

6 Results and Discussions

Fig. 2 shows the dynamic solution of the substrate concentration, S_1, when the total dilution rate, D ($D = 1/\tau$, in the two reactors was set at 0.17 h^{-1} with $D_1 = 0.25\ h^{-1}$, the fraction recycled, R = 0.35, $S_0 = 200$ mg/l, and the concentration factor, b = 1.9. It is apparent from the figure that self-sustained natural oscillation is attained within 100 hours of operation.

Simulations were carried out to investigate the effect of the dilution rate of the first reactor, D_1, on the overall system performance while keeping the overall dilution rate, D, constant. Fig. 3 shows the plot of the substrate concentration at discharge, S_2, (time-averaged or steady state) versus D_1 for different values of the fraction recycled while keeping the total dilution rate fixed at 0.17 h^{-1}. It shows that with the variation of the dilution rate the operating state of the reactor system changes. When the dilution rate of the first reactor was kept low (for example, $D_1 = 0.2\ h^{-1}$), it was observed that the reactor system operates at oscillatory state. However, at some intermediate values of D_1 (for example, $D_1 = 0.25\ h^{-1}$ for recycle ratio of 0.45) the system changes from oscillatory state to steady state. When D_1 was increased further to a value of about 0.4 h^{-1} for the same recycle ratio, the operation of the system changes again to oscillatory state. The occurrence of the second oscillatory region is because of the recycle of effluents from the second reactor to the first reactor. It is likely that either the first reactor or the second reactor operates at oscillatory state for a set of process variables when the total dilution rate, D, is kept constant. Then, even if the first reactor operates at steady state for the choice of process parameters, it inherits forced oscillation through recycle of oscillatory-state operation of the second reactor. From Fig. 3, it is also apparent that the switch from oscillatory state to steady state to oscillatory state occurs at different values of the dilution rate, D_1, and for different values of the recycle ratio, R. When the fraction recycled is less than about 0.3, the overall system is always under oscillatory state. However, the fraction recycled is increased, the overall system does not always operate under oscillatory state. For example, when the fraction recycled is equal to 0.4, no limit cycle exists for D_1 between 0.26 and 0.4 h^{-1}. Fig. 3 also reveals that when the first reactor operates under oscillatory state the concentration of the substrate at discharge is lower than when the second reactor is operating under oscillatory state. This is probably because only a fraction of the effluent from reactor 2 was recycled to the first reactor. This is apparent from the figure as the difference decreases with the increase of the recycle ratio. Fig. 3 further divulges that the concentration of the substrate at discharge from the second reactor decreases with the increase of the recycle ratio. Therefore, it is better to operate at a high recycle ratio although it will increase the operating cost. A close scrutiny of the figure shows that when the fraction recycled is less than 0.41, operation with oscillatory state will enhance the performance of the reactor over steady-state operation, as the substrate concentration at discharge is lower. This is shown in Fig. 4 where for each particular recycle ratio the best operating parameter was chosen, i.e., the best D_1 for a given D. In essence, Figs. 3 and 4 are plots of the parameter space for the distinguished bifurcation parameters D_1 and R showing a range of values of D_1 and R for fixed values of D for which whether oscillatory state or single steady state exist, and whether performance improvement of the overall system can be achieved by operating reactors in one particular mode over the other. Similar plots with different values of the process parameters (such as τ, S_0, X_0, B_0, P_0, H_0, b, etc.) or kinetic parameters (such as μ_m, K, and Y) can be obtained to determine the regions where oscillatory-state operation will be advantageous over steady-state operation.

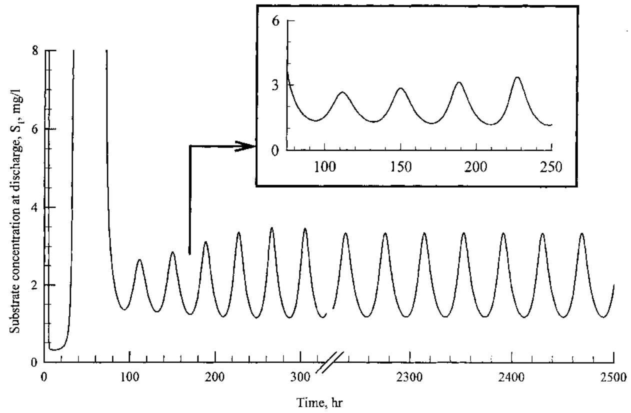

Figure 2. Self-sustained natural oscillation (limit cycles). Reference values: D = 0.17 h^{-1}, S_0 = 260 mg/L, B_0 = 30 mg/l, b = 1.9, R = 0.35, D_1 = 0.25 h^{-1}.

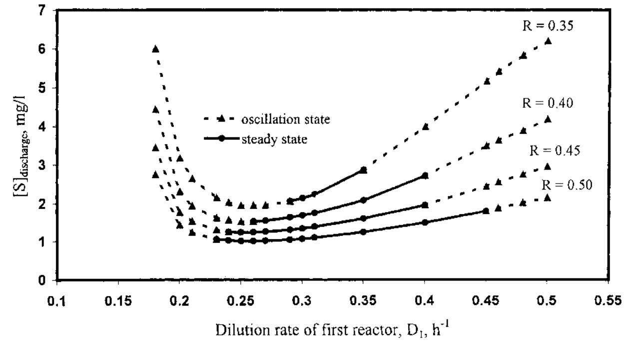

Figure 3. Effect of dilution rate of the first reactor, D_1, on overall system performance for different recycle ratio, R. Reference values: D = 0.17 h^{-1}, B_0 = 30 mg/l, S_0 = 260 mg/l, b = 1.9.

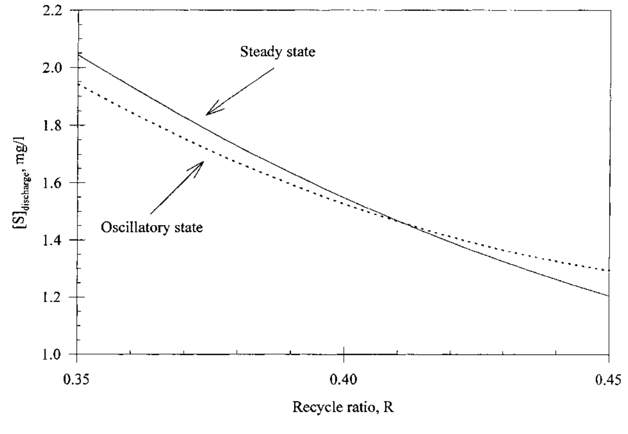

Figure 4. Effect of recycle ratio, R, on overall system performance. Reference values: D = 0.17 h^{-1}, B_0 = 30 mg/l, S_0 = 260 mg/l, b = 1.9.

From the above discussions it is evident that other parameters (dilution rate, fraction recycled, sewage concentration, and concentration factor) will have a similar effect on the substrate concentration at discharge and depending on the parameter values, only steady state or only oscillatory state or both states can exist. The system may operate at steady state or at oscillatory state. For some parameter values, steady-state operation will be better in terms of reactor performance, while for some other combination of parameters, oscillatory-state operation yields a better reactor performance. A comprehensive study of the effects of all process parameters on the reactor performance will reveal the regions where oscillatory-state condition exists. One may operate the reactor at these regions for economic benefits or if intended, may avoid them completely in practice. Therefore, as an engineer, one should be prepared to utilize these situations for economic benefits, or at least, should know how to avoid them in practice.

Tab. 2 summarizes all the simulation runs and depicts the region of values of the recycle rate, R, for which a reactor performance improvement is possible when operated under oscillatory condition. The last column of the table lists the values of the recycle rate below which operating the reactor under oscillatory state will yield a better reactor performance. Therefore, if one wants to improve the reactor performance, he/she must operate the reactor below the listed value of R. However, if one wants to avoid the reactor being operated under oscillatory conditions, he/she must select operating parameters such that steady state prevails in the reactor. The arrow (↑or ↓) means whether the value used was greater than or less than the reference set of values. From the table it is apparent that the region for which performance improvement can be achieved under oscillatory condition will increase if the values of the sewage bacteria concentration, B, and the substrate concentration, S, are increased over the reference set of values, while the values of the total dilution rate, D, and the concentration factor, b, are decreased over the reference set of values. To ascertain the above fact that the oscillatory region is extended (i.e., the range of the recycle ratio) while improving the reactor performance, the parameters were selected as follows: $D = 0.10\ h^{-1}$, $B_0 = 40$ mg/l, $S_0 = 350$ mg/l, and b = 1.5. However, it was observed that for the selected process parameters as above, oscillation existed for the entire region (i.e., for all values of the recycle ratio, R). This means that for the choice of the above set of parameter values, oscillation will always prevail, simple steady state will not occur. Therefore, there must be a set of parameter values for which the entire parameter space can be divided into two regions, one where only oscillatory state exists, and the other where the existence of both steady state and oscillatory state is possible depending on the fraction recycled employed.

Table 2. Summary of results. Regions in parameter space where performance improvement can be achieved by natural oscillation.

Parameters	D (h^{-1})	B_0 (mg/l)	b	S_0 (mg/l)	Improvement by oscillation *when*
Run 1 (Ref)	0.17	30	1.9	260	R < 0.41
Run 2	0.19 ↑	30	1.9	260	R < 0.2
Run 3	0.15 ↓	30	1.9	260	R < 0.65
Run 4	0.17	35 ↑	1.9	260	R < 0.57
Run 5	0.17	25 ↓	1.9	260	R < 0.2
Run 6	0.17	30	2.1 ↑	260	R < 0.27
Run 7	0.17	30	1.7 ↓	260	R < 0.6
Run 8	0.17	30	1.9	300 ↑	R < 0.43
Run 9	0.17	30	1.9	220 ↓	R < 0.35

The most important parameters which are important in the present study are the total dilution rate, D, the sewage bacteria concentration, B_0, and the substrate concentration, S_0. At first, the parameter space was determined for D and B_0, keeping the values of S_0 and b fixed at 350 mg/l and 1.5 respectively. Fig. 5 shows the region where only oscillatory state exists (region below the line) and the region where oscillatory state or steady state can occur depending on the values of D_1 and R (region above the line). When the operating parameters (D and B_0) are chosen such that one is in the region above the line, it is possible to find a suitable combination of parameter values (D_1 and R) such that steady state exists for choice of a certain combination, while oscillatory state exists for some other choice of parameter values. It would then be possible to compare the two states to determine whether performance improvement is likely by operating the reactor under oscillatory state.

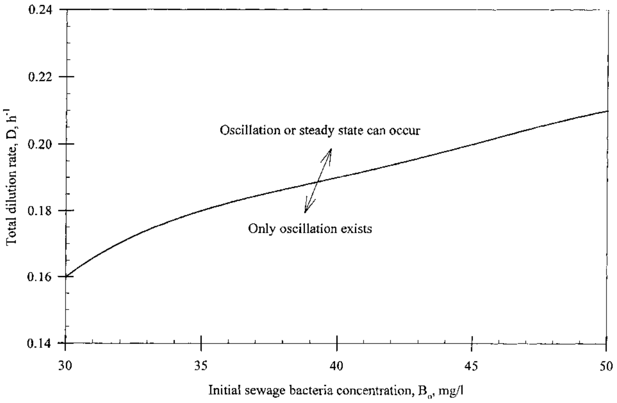

Figure 5. Effect of total dilution rate, D, and initial sewage bacteria concentration, B_0, on overall system performance. Reference values: $S_0 = 350$ mg/l, b = 1.5.

To determine the extent of performance improvement possible in such case, we selected the following set of parameter values: $D = 0.20\ h^{-1}$, $B_0 = 40$ mg/l, $S_0 = 350$ mg/l, and b = 1.5. Fig. 6 shows the substrate concentration at discharge when the above process parameters were selected. The figure reveals that when the recycled rate is greater than 0.7 only steady state prevails, while only oscillatory state prevails, when the fraction recycled is less than 0.3. In addition, a better reactor performance is observed when the reactor is operated at steady state for a recycle ratio between 0.65 and above, while oscillatory operation improves the reactor performance when the fraction recycled is below 0.65. From the figure it can be seen that the biggest improvement in this case is obtained when the recycled rate is 0.3. The substrate concentration at discharge is decreased from 10.64 mg/l to 9.02 mg/l, an increase of 15.23 %. It is to be noted that the improvement in conversion was achieved in the region (parameter space) where both steady state and oscillatory state exist. In fact, oscillatory operation is better when R is less than 0.65 and oscillatory-state operation is the only possibility when R is less than 0.3.

A similar effect can be studied for D and S_0, keeping the values of B_0 and b fixed at 40 mg/l and 1.5 respectively. Fig. 7 shows such a plot of the parameter space for D and S_0, keeping the values of B_0 and b fixed at 40 mg/l and 1.5 respectively. The figure shows the region where only oscillatory state exists (region below the line) and the region where oscillatory state or steady state can occur depending on the values of D_1 and R

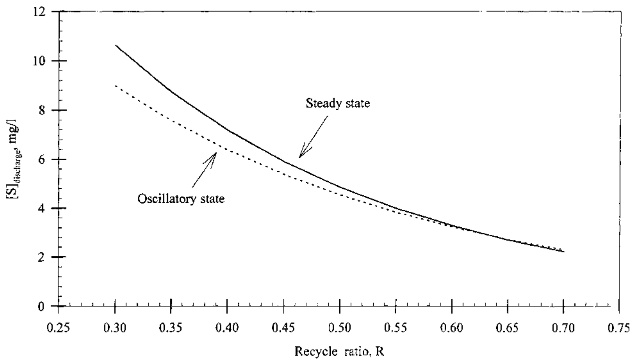

Figure 6. Effect of recycle rate, R, on overall system performance. Reference values: D = 0.2 h^{-1}, B_0 = 40 mg/l, S_0 = 350 mg/l, b = 1.5.

(region above the line). When the operating parameters (D and S_0) are chosen such that we are in the region above the line, then it is possible to find a suitable combination of parameter values of D_1 and R such that steady state exists for choice of a certain combination, while oscillatory state exists for some other choice of parameter values. However, for this case, it is apparent from the figure that the regions are quite insensitive to the values of the total dilution rate, D. Only oscillatory state exists whenever the total dilution rate, D, is below about 0.2 h^{-1}, while both states are possible if D is greater than 0.2 h^{-1}. To determine the extent of performance improvement possible in this case, we selected the following set of parameter values: D = 0.25 h^{-1}, B_0 = 40 mg/l, S_0 = 500 mg/l, and b = 1.5. However, it was observed that steady-state operation is always better than oscillatory-state operation.

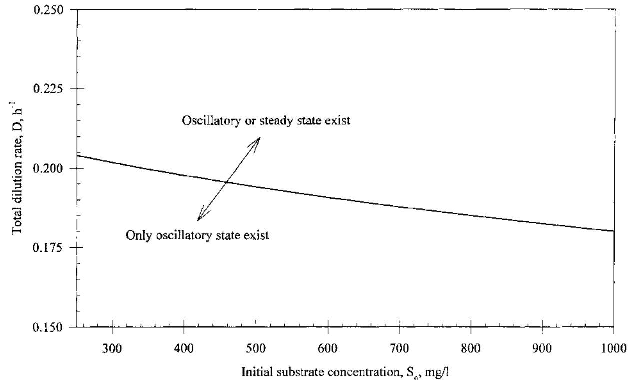

Figure 7. Effect of total dilution rate, D, and initial substrate concentration, S_0, on overall system performance. Reference values: B_0 = 40 mg/l, b = 1.5.

7 Conclusions

A novel operation strategy is used to improve the overall reactor performance of the activated sludge wastewater treatment process. If the process parameters are selected properly, it is possible to generate self-sustained free oscillation within a reactor that can then act as a forcing system for other reactors in series. In experimental studies and in eventual application to industrial processes, this system is advantageous in that no external energy is required to generate this situation. The improved performance is accomplished at no additional costs, an attractive proposition to the cost-conscious processing industry. Even though the novel operation strategy was applied to the activated sludge wastewater treatment process, it represents nevertheless a broad range of physical systems for chemical reactors and is very rich in the variety of dynamical behaviors that can occur. The mathematical treatment can be easily extended for other cases, although it is possible that additional new complexities may arise. The controversial question of whether dynamic operation of a chemical reactor under oscillatory conditions is economically beneficial or not over the steady state result is not answered in this paper. However, we should emphasize that oscillations have been reported for systems in all areas of nonlinear dynamics, and the present system is by no means an exception. Therefore, as an engineer, one should be prepared to utilize these situations for economic benefits, or at least should know how to avoid them in practice.

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Symbols used

A	[–]	Jacobian matrix
b	[–]	concentration factor
B	[mg/l]	concentration of sewage bacteria
C	[mg/l]	total concentration of protozoa
D	[h^{-1}]	dilution rate
F	[m^3/s]	flow rate
H	[mg/l]	concentration of free-swimming ciliate in reactor
K	[mg/l]	saturation constant
p	[–]	operating parameter
P	[mg/l]	concentration of attached ciliate in reactor
R	[–]	fraction recycled
S	[mg/l]	soluble substrate concentration
t	[h]	time
V	[m^3]	reactor volume
x	[–]	state variable
X	[mg/l]	concentration of sludge bacteria in reactor
Y	[–]	yield constant of the microorganism

Greek symbols

λ	[–]	eigenvalue
μ	[h^{-1}]	specific growth rate of microorganism
μ_m	[h^{-1}]	maximum specific growth rate of the microorganism
σ	[–]	bifurcation parameter

Subscripts

1	reactor 1
2	reactor 2
0	inlet, initial
s	steady state
w	wastage

References

[1] Douglas, J.; Rippin, D., Unsteady-State Process Operation, *Chem. Eng. Sci. 21* (1966) p. 305.

[2] Cutlip, M., Concentration Forcing of Catalytic Surface Rate Processes, *AIChE J. 25* (1979) p. 502.

[3] Wilson, S.; Rinker, R., Concentration Forcing in Ammonia Synthesis, *Chem. Eng. Sci. 37* (1982) p. 343.

[4] Lee, C.; Bailey, J., Modification of Reaction Selectivity by Periodic Operation, *Ind. & Eng. Chem. Proc. Des. Dev. 19* (1980) p. 160.

[5] Lee, C.; Yeung, S.; Bailey, J., , Experimental Studies on Forced Periodic CSTR, *Can. J. Eng. 58* (1980) p.12.

[6] Bailey, J., *Periodic Phenomena in Chemical Reactor*, Chap. 12, Prentice-Hall 1977, p. 758.

[7] Ray, A. K., Performance Improvement of a Chemical Reactor by Natural Oscillations, *Chem. Eng. J. 59* (1995) p. 169.

[8] Golubitsky, M.; Schaeffer, D., *Singularities and Bifurcation Theory*, vol. 1, Springler, New York 1985.

[9] Uppal, A.; Ray, W.; Poore, A, On the Dynamic Behaviour of CSTR, *Chem. Eng. Sci. 29* (1974) p. 967.

[10] Ganczarczyk, J. J., *Activated Sludge Processes: Theory and Practice*, Marcel Dekker Inc., New York 1983.

[11] Pavlou S., Dynamics of a Chemostat in Which One Microbial Population Feeds on Another, *Biotechnol. and Bioeng. 27* (1985) p. 1525.

[12] Fujie, K.; Tsubone, T.; Moriya, H.; Kubota, H, A Simplified Kinetic Model to Simulate Soluble Organic Substances Removal in an Activated Sludge Aeration Tank, *Water Research 22* (1988) 1, p. 29.

[13] Padukone, N.; Graham F. A., A Simple, Conceptual Mathematical Model for the Activated Sludge Process and Its Variants, *Water Research 23* (1989) 12, p.1535.

[14] Curds, C. D., A Computer Simulation Study of Predator-Prey Relationships in a Single-Stage Continuous-Culture System, *Water Research 5* (1971a) p. 793.

[15] Curds, C. D., Computer Simulations of Microbial Population Dynamics, *Water Research 5* (1971b) p. 1049.

[16] Vasiliev, V. B.; Vavilin, V. A., Model of Biological Treatment With Sequential Pollutant Oxidation, *Biotechnol. and Bioeng. 27* (1985) p. 490.

[17] Braha, A.; Hafner, F., Use of Monod Kinetics on Multi-Stage Bioreactors, *Water Research 19* (1985) p. 1217.

[18] Henze, M., A General Model for Single-Sludge Wastewater Treatment Systems, *Water Research 21* (1987) 5, p. 505.

[19] Lessard, P.; Beck, M. B., Dynamic Modelling of the Activated Sludge Process: a Case Study, *Water Research 27* (1993) 6, p. 963.

[20] Cote, M.; Grandjean, B. P. A.; Lessard, P.; Thibault, J., Dynamic Modeling of the Activated Sludge Process Improving Prediction Using Neural Networks, *Water Research 29* (1995) 4, p. 995.

[21] Daigger, G. T.; Nolasco, D., Evaluation and Design of Full-Scale Wastewater Treatment Plants Using Biological Process Models, *Water Sci. and Technol. 31* (1995) 2, p. 245.

[22] Curds, C. D., A Theoretical Study of Factors Influencing the Microbial Population Dynamics of the Activated-Sludge Process – I, *Water Research 7* (1973) p. 1269.

[23] Himmelblau, D. M., *Basic Principles and Calculations in Chemical Engineering*, 6th ed., Prentice-Hall, Upper Saddle River, New Jersey 1996.